Sub-Poisson current noise in ballistic space-charge-limited diodes with linear $I$–$V$ characteristics

O. M. Bulashenko and J. M. Rubí
Departament de Física Fonamental, Universitat de Barcelona, Diagonal 647, E-08028 Barcelona, Spain

V. A. Kochelap
Department of Theoretical Physics, Institute of Semiconductor Physics, Kiev 252028, Ukraine

(Received 22 July 1999; accepted for publication 29 August 1999)

It is generally believed that the $I$–$V$ characteristics for a ballistic space-charge-limited (SCL) diode exhibit the Child law $I\propto V^{3/2}$. In this letter, we present the exact formulas which describe the $I$–$V$ characteristics and the current noise (temperature) in a semiconductor ballistic SCL diode from which follows the linear or sublinear $I$–$V$ dependencies, while the noise is strongly suppressed by Coulomb correlations below the Poissonian level. © 1999 American Institute of Physics.

[0003-6951(99)04143-1]

In general, microdevice performance is affected stronger by electronic noise as the device dimensions are scaled down. To overcome this problem, several intrinsic mechanisms of limitation and suppression of noise are nowadays widely discussed. In nanostructures, reduction in the shot noise occurs due to the Pauli exclusion principle or Coulomb correlations. The latter noise-reduction effects are especially important, since the shot noise is a residual noise even at zero temperature and in the absence of random scattering processes in ballistic samples. The continuing scaling down of practical devices will inevitably lead to such short-channel devices where the transport is ballistic.

In this letter, we address the noise-reduction effect caused by the Coulomb correlations of injected carriers in a ballistic space-charge-limited (SCL) diode. From the theory of a vacuum diode, it is known that the thermionic emission of electrons from a hot cathode results in a space charge which is manifested in the nonlinearity of the $I$–$V$ characteristics, described usually by the Child–Langmuir theory, and in the shot-noise reduction, described by the North’s theory. The similar effects are currently widely discussed in respect to their importance in solid-state ballistic devices. Below, we demonstrate that the known formulas for vacuum electronics may not be applied for a two-terminal semiconductor SCL diode at biases, and for diode lengths, which are relevant for the ballistic transport regime. To this purpose, we propose the closed analytical formulas to evaluate exactly the $I$–$V$ characteristics and spectral densities of suppressed noise in these devices. The formulas account for the Poissonian injection from both terminals and allow one to discuss the noise reduction in terms of a measurable quantity—the effective noise temperature.

Consider a two-terminal semiconductor ballistic sample with plane-parallel heavily doped contacts at $X=0$ and $X=d$, which we denote by $L$ and $R$ (Fig. 1). The applied bias $U$ between the contacts is assumed to be fixed by a low-impedance external circuit. The contacts are assumed to be heterojunctions, forming with the base a $n$-$i$-$n$ heterodiode operating under the current injection regime. The Fermi level in the contacts is sufficiently below the edge of the conduction band in the base, so that the injected distribution function is nondegenerate. Let the space charge be such that a potential minimum $\psi_m=V_m$ occurs at $x=x_m$, which acts as a barrier for the electrons by reflecting a part of them back to the contacts. We use the potential $\psi$ in units of $k_BT/q$, the electron density in units of the density $N_0$ injected from each contact, and the coordinate $x$ is scaled by the Debye screening length $L_D=\sqrt{k_BT/(2q^2N_0)}$. In such units the results will be determined by two dimensionless parameters: (i) the diode length (or the screening parameter) $\lambda=d/L_D$, and (ii) the applied voltage bias $V=qU/(k_BT)$.

The electron density distribution in the ballistic base may be obtained in a similar form as for the vacuum diode case. By making use of the Maxwellian injection from two contacts and the contribution of different groups of carriers (transmitted and reflected), one gets

$$n(\eta)=n_m e^{\eta[1 \pm \beta \text{erf}(\sqrt{\eta})]},$$

where $\eta(x)=\psi(x)-\psi_m$ is the shifted potential measured from the minimum, $\text{erf}$ stands for the error function, $n_m=\frac{1}{2}e^{-V_m}(1+e^{-V})$ is the electron density at the potential minimum, and $\beta=\tanh(V/2)$. Here, the upper sign applies...
for the left side of the potential minimum $0 < x < x_m$, and the lower sign applies for the right side $x_m < x < \lambda$. If in Eq. (1) we set $\beta = 1$, we obtain the well-known formula for the vacuum diode with a single-injection contact. Equation (1) may, therefore, be viewed as an extension of the single-injection Langmuir theory to the double-injection case.

The steady-state current, consisting of two opposing injection currents, is given by

$$I = I_c e^{-\frac{V_m}{V}} \left[ 1 - e^{-\frac{V}{V_m}} \right] = 2 I_c n_m \beta,$$

(2)

where $I_c = q N_0 / QA$ is the emission current from each contact, and $\bar{V} = \frac{1}{2} k_B T (\pi m)^{1/2}$ is the average velocity of injected electrons. The potential minimum $V_m$ we find by solving the Poisson equation $d^2 \eta / dx^2 = n(\eta)$ together with Eq. (1), and one gets

$$\lambda \sqrt{2 n_m} = \int_0^{V_m} \frac{d \eta}{\sqrt{\bar{V}(\eta)}} + \int_{V_m}^{V} \frac{d \eta}{\sqrt{\bar{V}(\eta)}},$$

(3)

$$h_{\bar{V}}(\eta) = e^{\eta} - 1 \pm \beta \left( e^{\eta} \text{erf} \sqrt{\eta} - \frac{2}{\sqrt{\pi}} \sqrt{\eta} \right),$$

(4)

where $\beta$ and $n_m$ are functions of $V$ as specified above.

The obtained Eqs. (2)–(4) determine the current–voltage characteristics for a ballistic two-terminal SCL diode. Again, the difference with the vacuum diode case is in the factor $\beta$ which drastically changes the behavior. In Fig. 2(a), we present the $I$–$V$ curves for different levels of screening $\lambda$. It is seen that the diode behaves as a linear resistor at low biases for all $\lambda$ despite the fact that the transport is space-charge limited. The curves for this case are described by $I = I_c V e^{-\frac{V}{V_m}}$, where $V_m^0$ is the equilibrium value of the potential minimum whose value depends on $\lambda$. In the range $1 \leq V' \leq 10$, the $I$–$V$ curves deviate to sublinear dependence [see the small-signal conductance $g = dI/dV$ in Fig. 2(b)]. Finally, at $V' = V_{cr}$, when the potential minimum vanishes, the current saturates at the value $I = I_c$. For short diodes, $\lambda \leq 10$, $V_{cr} \leq 10$, the sublinear curves end up by the saturation regime. With increasing the diode length $\lambda$, the critical voltage increases, and the sublinear dependence changes to the superlinear one. The latter should recover the Child law $I \propto V^{3/2}$ in the asymptotic limit $\lambda \rightarrow \infty$, $V \rightarrow \infty$, satisfying simultaneously $V' < V_{cr}$.

At this point, we would like to emphasize the important difference between semiconductor and vacuum ballistic diodes. In vacuum diodes the applied voltage may be quite large without breaking down the ballistic transport regime. To the contrary in solids, electrons, even for a pure material, interact with the lattice. At high biases, this interaction becomes quite strong due to the significant increase of the electron energy. Thus, one cannot bias the sample to the voltage more than, for instance, the threshold for the optical phonon generation, since a strong interaction with the lattice will break down the ballistic regime. The allowed range of biases is typically restricted by $U \leq 50 k_B T / q$. Then, for real structures the ballistic lengths $\lambda$ are well below 100. It is clear from Fig. 2(a) that in this range the asymptotic Child limit is not achieved. This means that for semiconductor ballistic SCL diodes one should use the more general formulas (2)–(4) from which follows the linear or sublinear $I$–$V$ dependences in a wide range of biases, even under a strong limitation of transport by a space charge. Finally, we have checked that the obtained solutions excellently correspond to the Monte Carlo simulations [the case of $\lambda = 30.9$ is compared in Fig. 2(a)].

For the spectral density of current fluctuations in the low-frequency regime (neglecting the transit time effects), we obtain analytically as an integral over the energy $\varepsilon$ of electrons injected from both $L$ and $R$ contacts

$$S_I = 2 q I_c e^{-\frac{V_m}{V}} \sum_{k=L,R} \int_{-\infty}^{\infty} \gamma_k(\varepsilon) e^{-\varepsilon} d\varepsilon,$$

(5)

$$\gamma_k(\varepsilon) = \begin{cases} -\frac{\beta}{2 \Delta_m} \int_{-\infty}^{\eta_k} \frac{G(\eta, \varepsilon)}{[h_{\bar{V}}(\eta)]^{1/2}} d\eta, & \varepsilon < 0, \\
1 - \frac{\beta}{2 \Delta_m} \left[ \int_{0}^{\eta_k} \frac{H(\eta, \varepsilon)}{[h_{\bar{V}}(\eta)]^{1/2}} d\eta + \int_{0}^{\eta_{\lambda}} \frac{H(\eta, \varepsilon) - \varepsilon}{2 \sqrt{\pi}} d\eta \right], & \varepsilon > 0, \\
0, & \varepsilon = 0, \\
\frac{\gamma_L(\varepsilon) - \gamma_R(\varepsilon)}{2 \Delta_m} e^{-\frac{V_m}{V}} [h_{\bar{V}}(\eta_{\lambda})]^{-1/2}, & \varepsilon < 0, \\
\frac{\gamma_L(\varepsilon)}{2 \Delta_m} - 2, & \varepsilon > 0, \end{cases}$$

(7)

where $\Delta_m = \sqrt{n_m^{1/2} + [h_{\bar{V}}(\eta_{\lambda})]^{1/2} + [h_{\bar{V}}(\eta_{\lambda})]^{-1/2},}$ $\eta_L = V_m^0$, $\eta_R = V_m + V$, $H(\eta, \varepsilon) = (2/\sqrt{\pi})(\sqrt{\eta + \varepsilon} - \sqrt{\varepsilon}),$ $G(\eta, \varepsilon) = (4/\sqrt{\pi})(\sqrt{\eta + \varepsilon} - \sqrt{\varepsilon}).$ To find the above expressions, we have solved the linearized Poisson equation by making use of the analytical method described in Ref. 13. Equation (5) allows one to obtain the current-noise spectral density for the given length of the conductor $\lambda$ and applied voltage $V$ in a full range of biases from thermal to shot noise limits under space-charge-limited transport conditions. Note that formulas (5)–(7) may also be applied for a vacuum diode by setting $\gamma_L = 0$ and $\beta = 1$.
The obtained current-noise spectral density \( S_I \), which accounts for the long-range Coulomb correlations, should be compared with the uncorrelated value \( S_I^{uncor} \), in order to see the noise suppression magnitude [see Fig. 2(b)]. At low biases, the current noise is independent of voltage and corresponds to the thermal equilibrium Nyquist noise described by the formula

\[
S_I = 4qI e^{-\frac{V}{V_T}} = 4k_B T g_0,
\]

where \( g_0 = dI/dV \big|_{V \to 0} \) is the zero-bias small-signal conductance. It is seen from Fig. 2(b), that if the screening parameter \( \lambda \) is sufficiently large, the noise level is substantially reduced at \( 1 \leq V < V_{cr} \). At \( V \geq V_{cr} \), the full shot noise level \( 2qI \) is abruptly recovered. The results are seen to be in excellent agreement with the Monte Carlo simulations.

It is interesting to see from Fig. 2(b) that in the limit of strong screening \( \lambda \to \infty \), despite the strong nonlinearity, the ratio between \( S_I \) and the small-signal conductance \( g \) tends to the constant value independent of the bias. It is reasonable, therefore, to characterize the noise properties of the diode through the effective noise temperature \( T_n = S_I / (4k_B g) \).

Figure 2(c) shows \( T_n \) versus the applied bias \( V \) for various \( \lambda \) calculated from the above equations. One can see that starting from \( T_n = T \) at zero bias it drops at \( V \approx 1 \) below the temperature of the injected electrons. \( T_n \) is a measurable quantity, and the observation of \( T_n < T \) would indicate the significance of the Coulomb correlation effects which suppress the current noise. At \( V \approx V_{cr} \), \( T_n \) is seen to sharply increase due to the current saturation (\( g = 0 \)), which may also be detected in the experiment. The minimal value of \( T_n \) in the limit \( \lambda \to \infty \), \( V \to \infty \) is given by

\[
T_n \approx 3(1 - \pi/4) \approx 0.644.
\]

It is seen from Fig. 2(c) that this value may not be achieved if the diode length (screening parameter) is not sufficiently large (\( \lambda \leq 30 \)).

To illustrate the results, consider the heterodiode with GaAs contacts and a \( \text{Al}_{0.05}\text{Ga}_{0.95}\text{As} \) ballistic base. For the contact doping \( 4 \times 10^{18} \text{ cm}^{-3} \) at \( T = 50 \text{ K} \), we obtain \( N_0 = 7.25 \times 10^{14} \text{ cm}^{-3} \) and \( L_D = 46 \text{ nm} \). By using Fig. 2(b), we find that for the 460-nm-length diode (\( \lambda = 10 \)) the minimal noise occurs at the bias \( \approx 25 \text{ mV} \), for which \( S_I / S_I^{uncor} \approx 0.2 \), i.e., current noise five times less than the Poissonian level. The full-shot noise recovers at 36 mV. For the 1.5 \( \mu \)-length diode (\( \lambda \approx 30 \)) the Poisson noise is suppressed as much as \( \approx 25 \) times.

In conclusion, strong suppression of the current noise in a ballistic SCL diode with almost linear \( I - V \) characteristics has been demonstrated, which may be of interest from the point of view of possible applications. We have proposed closed analytical expressions to evaluate the steady-state current, the current noise spectral density, and the noise temperature under the full range of applied biases in this device. These formulas may be used to estimate the importance of the noise suppression effects in small-size two-terminal ballistic devices.

This work has been supported by the Generalitat de Catalunya, Spain and NATO linkage Grant No. HTECH.LG 974610.

11. D. O. North, RCA Rev. 4, 441 (1940).
14. \( k_B T_n \) has a meaning of the maximum noise power per unit bandwidth which can be delivered to an output matched circuit, thus it is a measurable quantity.